Mini-Bibliography... Apportionment

by Joseph Malkevitch

The apportionment problem belongs to the part of mathematics which deals with fairness questions. As a simple example, suppose that a modest sized corporation decides to add 22 employees. The corporation has 4 divisions, which account for 18, 23, 29, and 30 percent of the corporation's business, respectively. How many of the new employees should be assigned to each division? The division with 30 percent of the business has as its "fair share" .30 of 22 or 6.6! Since a fraction of a person can not be assigned to a division, how is the problem to be resolved? This same question arises in many settings, including how to assign states to seats in the House of Representatives based on their populations (see Mathematics in the News in this issue) or the apportioning of seats to different parties in European Parliamentary democracies based on the portion of the vote that each party received. The crux of the problem deals with the fact that an integer must be written as the sum of other integers, rather than the sum of decimals!

Balinski, M. and H.P. Young, ***Fair Representation: Meeting the Ideal of One Man, One Vote,*** *Yale University Press, 1982.* This is the definitive book on the apportionment problem. It is highly readable and includes a detailed historical account of the different procedures and approaches to apportioning the House of Representatives (US). The story is wonderfully rich in history, involving such Americans as Alexander Hamilton, Thomas Jefferson, John Quincy Adams, and Daniel Webster. The mathematics appears in appendices at the end.

Bradberry, R., ***A Geometric View of Some Apportionment Paradoxes,*** *Math. Magazine, 65 (1992) 3-17.* A somewhat technical but illuminiating discussion of some of the unexpected phenomena associated with apportionment problems. It contains many diagrams to help understand small apportionment examples.

Eisner, M.J., ***Methods of Congressional Apportionment,*** *COMAP (Lexington, MA) Module, #620.* This is an example-driven, condensed account of apportionment written for college and high school students.

Lucas, W., ***The Apportionment Problem, in Political and Related Models,*** *S. Brams, W. Lucas, and P. Straffin, (eds.), Springer-Verlag, New York, 1978.* Excellent survey article in a wonderful book dealing with applications of mathematics in political science.

Rae, D., ***The Political Consequences of Electoral Law,*** *Rev. Ed., Yale University Press, 1971.* This unusual book attempts to account for differences in the stability of different countries based on the methods that are used to apportion their parliaments. (Emphasis is on European democracies.)

Robertson, J. et al., ***The Apportionment Problem: The Search for the Perfect Democracy,*** *HIMAP Module #8, COMAP, Lexington, MA 1986.* This a module that deals with different approaches to the apportionment problem, written for high school teachers and their students. Different concepts of equity are explored, the historical context mentioned, and various mathematical implementations are treated.

Saari, D., ***Apportionment Methods and the House of Representatives,*** *Amer. Math. Monthly, 85 (1978) 792-802.* A survey of mathematical ideas involving apportionment.

Steen, L., (ed.), ***For All Practical Purposes (2nd ed.),*** *W.H. Freeman, New York, 1991.* Chapter 11 has an elementary treatment of the apportionment problem.

Young, H.P. (ed.), ***Fair Allocation,*** *American Mathematical Society, Providence, 1985.* The volume is a collection of articles on fairness. It includes an article entitled the **Apportionment of Representation** by Balinski and Young. This is an expository treatment (though technical) of the work in their more technical papers, and includes an excellent bibliography. The remaining articles in this volume deal with fairness in taxes, auctions, and voting. ■

An Apportionment Problem *(Continued from page 8)*

We begin by giving each state one seat. To indicate this we have shaded the numbers in the top row. Now which state should get the next seat? If state B gets the 4th seat, there would be one representative for 85 people. This seems unfair since state A at this stage would then have only 1 seat for 740. Hence state A has the best claim to the next seat. We continue distributing seats in this manner until all 10 available seats are gone. Once A has 5 seats, it has one seat per 148 people, so the next seat, the 8th one would go to A. In the general case, the state having the next largest number in the table which is not yet shaded gets the next seat. In this situation, A gets 8 seats, B gets 1 seat, and C gets 1 seat (as witnessed by the shaded numbers in the table). A and C are over-represented and B is under-represented. Had there been an 11th seat to distribute, it would have gone to B rather than A, since in the table above 85 is bigger than 82.2. Other apportionment methods use similar priority tables but the divisors for each row are different from the 2, 3, 4, etc. used for the Jefferson method. ■

	A	B	C
P	740	170	90
P/2	370	85	45
P/3	246.6	56.7	30
P/4	185	42.5	22.5
P/5	148	34	18
P/6	123.3	28.3	15
P/7	105.7	24.3	12.9
P/8	92.5	21.2	11.3
P/9	82.2	18.9	10